SHEAR STRENGTH CRITERIA FOR ROCK, ROCK JOINTS, ROCKFILL, INTERFACES AND ROCK MASSES.

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Summary. Although many intact rock types can be very strong, a critical confining pressure can eventually be reached in triaxial testing, such that the Mohr shear strength envelope becomes horizontal. This critical state has recently been better defined, and correct curvature, or correct deviation from linear Mohr-Coulomb has finally been found.

Standard shear testing procedures for rock joints, using multiple testing of the same sample, in case of insufficient samples, can be shown to exaggerate apparent cohesion. Even rough joints do not have any cohesion, but instead have very high friction angles at low stress, due to strong dilation.

Great similarity between the shear strength of rock joints and rockfill is demonstrated, and the interface strength between rockfill and a rock foundation is also addressed.

Rock masses, implying problems of large-scale interaction with engineering structures, may have both cohesive and frictional strength components. However, it is not correct to add these, following linear Mohr Coulomb (M-C) or non-linear Hoek-Brown (H-B) standard routines. Cohesion is broken at small strain, while friction is mobilized at larger strain and remains to the end of the shear deformation. The criterion 'c *then* tan φ' should replace 'c *plus* tan φ' for improved fit to reality. In all the above, scale effects need to be accounted for.

Keywords. Rock, rock joints, rock masses, shear strength, friction, critical state, cohesion, dilation, non-linear, scale effects.

Introduction

Figure 1 illustrates a series of simple empirical strength criteria that pre-date Hoek-Brown, and that are distinctly different from linear-Mohr-Coulomb, due to their consistent non-linearity. Several of these categories will be addressed in this lecture and extended abstract.

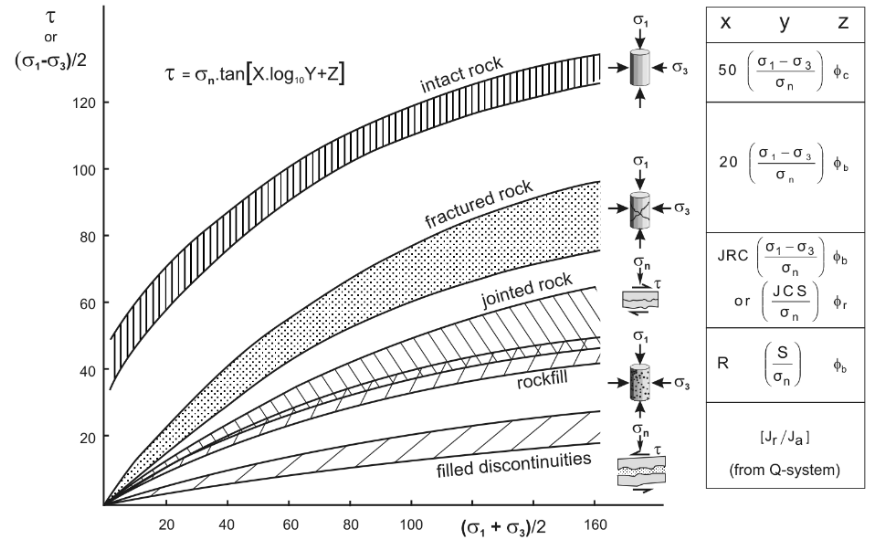

Fig. 1. *Simple empiricism, sometimes based on hundreds of test samples, suggested the following ways to express peak shear strength in rock mechanics and rock engineering. Note the general lack of cohesion. Derived from Barton, 1976, and Barton, 2006.*

Shear Strength of Intact Rock

The shear strength envelopes for intact rock, when tested over a wide range of confining stress, have marked curvature, and eventually reach a horizontal stage with no further increase in strength. This was termed the 'critical state' and the simple relation $\sigma_1 = 3\ \sigma_3$ suggested itself, as illustrated in Figure 2. Singh et al., 2011 have now modified the Mohr-Coulomb criterion by absorbing the critical state defined in Barton, 1976, and then quantified the necessary deviation from the linear form, using a large body of experimental test data.

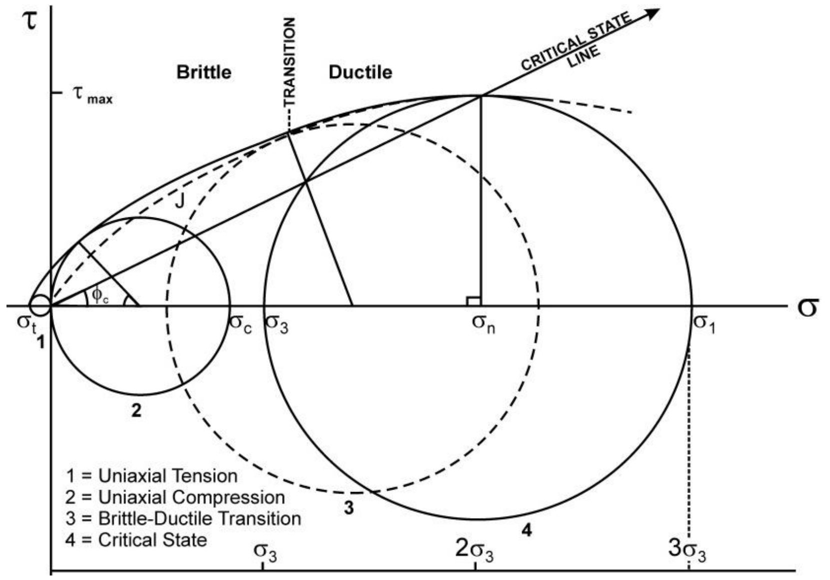

Fig. 2. *Critical state line defined by $\sigma_1 = 3\,\sigma_3$ was suggested by numerous high-pressure triaxial strength tests. Note the chance closeness of the unconfined strength (σ_c) circle to the confining pressure $\sigma_{3\ (critical).}$ Barton, 1976. Note that 'J' represents jointed rock. The magnitude of φ_c is 26.6° when $\sigma_1 = 3\,\sigma_3$.*

The Singh et al., 2011 development revealed the astonishing simplicity of the following equality: $\sigma_c \approx \sigma_{3\ (critical)}$ *for the majority of rock types*: in other words the two Mohr circles referred to in Figure 2 are usually touching at their circumference. The curvature of peak shear strength envelopes is therefore now more correctly described, so that few triaxial tests are required, and need only be performed at low confining stress, in order to delineate the whole strength envelope.

Shear Strength of Rock Joints

Figure 3 illustrates the non-linear form of the strength criterion for rock joints. It will be noted that no cohesion intercept is intended. A linear cut-off to the origin is used at very low stress, to represent the extremely high friction angles measured at low stress. It will be noted that subscripts have been added to indicate scale-effect (reduced) values of joint roughness JRC_n and joint wall strength JCS_n. This form is known as the Barton-Bandis criterion. Its effect on strength-displacement modelling is shown in Figure 4.

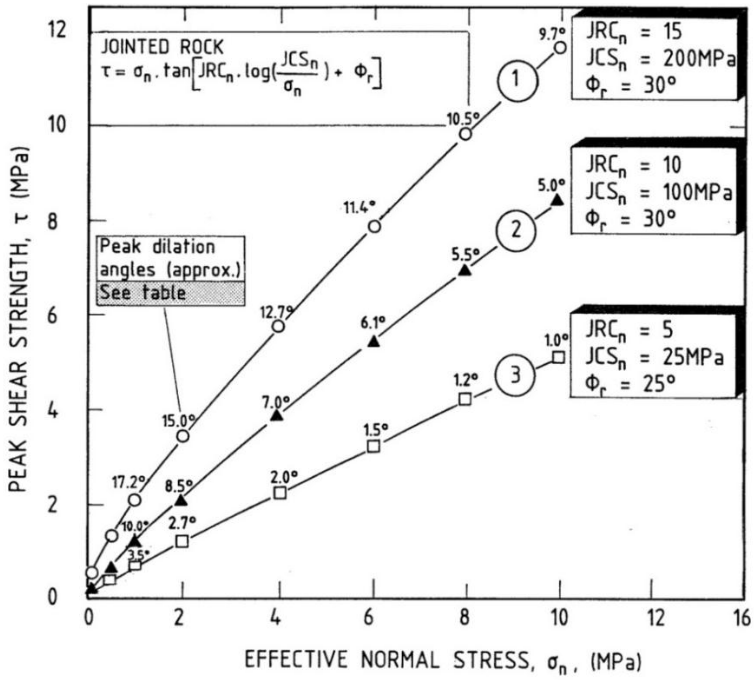

Fig. 3. *The scale-effect corrected form of the non-linear Barton 1973 strength criterion, following modification with φ_r by Barton and Choubey, 1977, and allowance for scale effects caused by block size. Note the strong dependence of dilation on joint properties.*

Shear Strength of Rockfill and Interfaces

Figure 1 showed that there were similarities between the shear strength of rockfill and that of rock joints. This is because they both have 'points in contact', i.e. highly stressed contacting asperities or contacting opposing stones. In fact these contacting points may be close to their crushing strength, such that similar shear strength equations can apply, as suggested in Figure 5

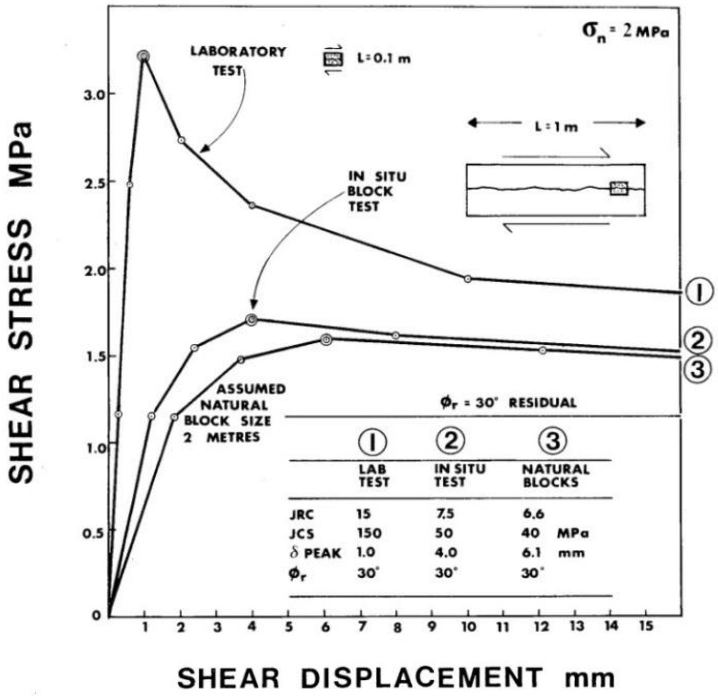

Fig. *4 Laboratory testing, especially of rough joints, may need a strong adjustment (down-scaling) for application in design, due to the block-size related scale effects on JRC and JCS. Barton, 1982.*

$\tau/\sigma_n = \tan\,[\mathrm{JRC}\ \log(\mathrm{JCS}/\sigma_n) + \varphi_r]$ applies to rock joints

$\tau/\sigma_n = \tan\,[\mathrm{R}\ \log(\mathrm{S}/\sigma_n) + \varphi_b]$ applies to rockfill

$\tau/\sigma_n = \tan\,[\mathrm{JRC}\ \log(\mathrm{S}/\sigma_n) + \varphi_r]$ might apply to interfaces

Because some dam sites in glaciated mountainous countries like Norway, Switzerland, and Austria have insufficient foundation roughness to prevent preferential shearing along the rockfill/rock foundation interface, artificial ‘trenching’ is needed. The preference for interface sliding (JRC-controlled) or failure within the rockfill (R-controlled) is illustrated in Figure 6.

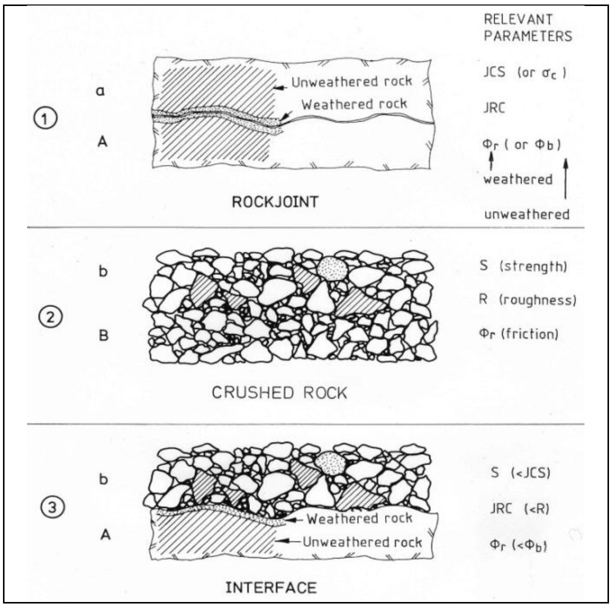

Fig. 5. *Peak shear strength estimates for three categories of asperity contact: rock joints, rockfill, and interfaces between the two.*

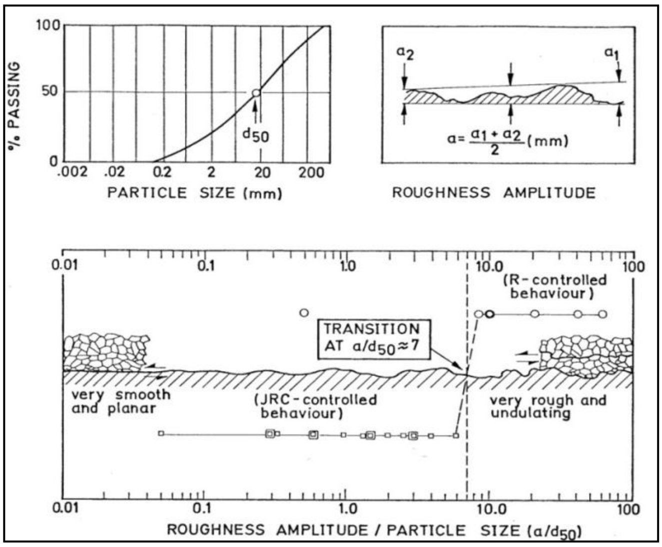

Fig. 6 *The results of interface/rockfill testing,showing R-controlled and JRC-controlled categories*

Shear Strength and Models of Rock Masses

It has been claimed – correctly – that rock masses are the single most complex of engineering materials utilized by man. The complexity may be due to variable jointing, clay-filled discontinuities, fault zones, anisotropic properties, and dramatic water inrush and rock-bursting stress problems. Nevertheless we have to make some attempt to represent this complexity in models. Two contrasting approaches (to simple cases) are shown in Figures 7 and 8.

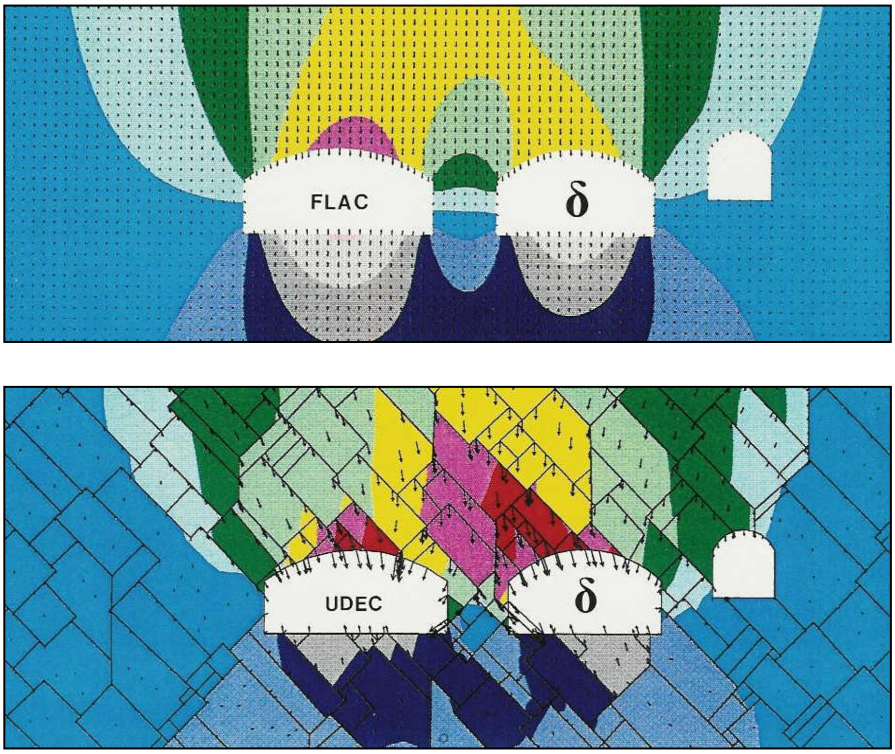

Fig. 7. *Continuum and discontinuum modelling approaches to the representation of tunnelling through an anisotropic rock mass. The increased richness and reality of representing the potential behaviour of jointing, even if exaggerated in 2D, is clear to see.*

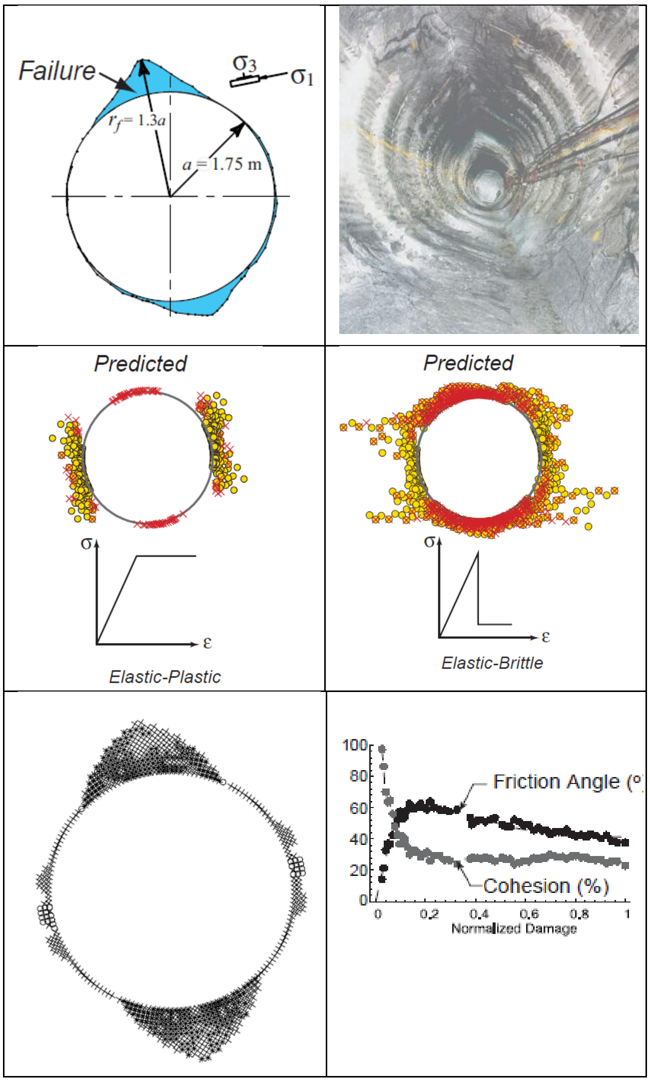

Fig.8. *Top: The Canadian URL mine-by break-out that developed when excavating by line-drilling, in response to the obliquely acting anisotropic stresses. This is followed by an important demonstration of unsuccessful modelling by 'classical methods' given by Hajiabdolmajid et al., 2000. They followed this with a more realistic degradation of cohesion and mobilization of friction in FLAC.*

The limitations of M-C, H-B and c plus σ_n tan φ

Attempts to model 'break-out' phenomena such as those illustrated in Figure 8, are not especially successful with standard Mohr-Coulomb or Hoek-Brown failure criteria, because the *actual phenomena* are not following our long-standing belief in *'c plus σ_n tan φ'*. The reality is degradation of cohesion at small strain and mobilization of friction (first towards peak, then towards residual) which occur at larger strain. The very important findings of Hajiabdolmajid et al., 2000 are summarised briefly by means of the six figures assembled in Figure 8. The demonstrated shortcomings of continuum modelling with *'c plus σ_n tan φ'* shear strength assumptions, should have alerted our profession for change already twelve years ago, but deep-seated beliefs or habits are traditionally hard to change.

Rock masses actually follow an even more complex progression to failure, as suggested in Barton and Pandey, 2011, who recently demonstrated the application of a similar *'c then tan φ'* modelling approach, but applied it in FLAC 3D, for investigating the behaviour of multiple mine-stopes in India. A further break with convention was the application of peak 'c' and peak 'φ' estimates that were derived directly from mine-logged Q-parameters, using the CC and FC parameters suggested in Barton, 2002. For this method, an estimate of UCS is required, as CC (cohesive component) and FC (frictional component) are derived from separate 'halves' of the formula for $Q_c = Q \times \sigma_c / 100$. See Table 1.

These much simpler Q-based estimates have the advantage of not requiring software for their calculation – they already exist in the Q-parameter logging data, and the effect of changed conditions such as clay-fillings, can be visualized immediately.

Table 1*. The remarkable complexity of the algebra for estimating c' and φ' with Hoek-Brown GSI-based formulations are contrasted with the simplicity of equations derived by 'splitting' the existing Q_c formula into two parts, as described in Barton, 2002.*
($Q_c = Q.\ \sigma_c/100$, with σ_{ci} expressed in MPa).

FC $$"\phi" \approx \tan^{-1}\left(\frac{J_r}{J_a} \times \frac{J_w}{1}\right)$$	(from Q)
$$\phi' = a\sin\left[\frac{6am_b\left(s+m_b\sigma'_{3n}\right)^{a-1}}{2(1+a)(2+a)+6am_b\left(s+m_b\sigma'_{3n}\right)^{a-1}}\right]$$	(from GSI)
CC $$"c" \approx \left(\frac{RQD}{J_n} \times \frac{1}{SRF} \times \frac{\sigma_c}{100}\right)$$	(from Q)
$$c' = \frac{\sigma_{ci}\left[(1+2a)s+(1-a)m_b\sigma'_{3n}\right]\left(s+m_b\sigma'_{3n}\right)^{a-1}}{(1+u)(2+a)\sqrt{1+\left(6am_b\left(s+m_b\sigma'_{3n}\right)^{a-1}\right)\Big/\left((1+a)(2+a)\right)}}$$	(from GSI)

Table 2. *Illustration of parameters CC (MPa) and FC^o for a declining sequence of rock mass qualities, with simultaneously reducing σ_c (MPa). Estimates of V_P (km/s) and E_m (GPa) are from Barton, 2002.*

RQD	Jn	Jr	Ja	Jw	SRF	Q	σ_c	Q_c	*FC*^{o}	*CC*	V_p	E_m
100	2	2	1	1	1	100	100	100	63	50	5.5	46
90	9	1	1	1	1	10	100	10	45	10	4.5	22
60	12	1.5	2	0.66	1	2.5	50	1.25	26	2.5	3.6	11
30	15	1	4	0.66	2.5	0.1	33	0.04	9	0.3	2.1	3.5

An important part of the verification of the mine stope modelling reported by Barton and Pandey, 2011 was the comparison of the modelling results with the deformations actually measured.

Table 3. Empirical equations linking tunnel or cavern deformation to Q-value, with span as input (left), and the ratio of vertical stress and UCS as additional input (right). From Barton, 2002. (Note: In left equation Δ is in mm, while span remains in meters, as in left axis of Figure 9. In right equation only: Δ mm, span mm, stress and strength in consistent units, e.g. MPa).

$$\Delta = \frac{SPAN}{Q}$$ (central trend of all data: approx)	$$\Delta_v = \frac{SPAN}{100Q}\sqrt{\frac{\sigma_v}{\sigma_c}}$$ (more accurate estimate)

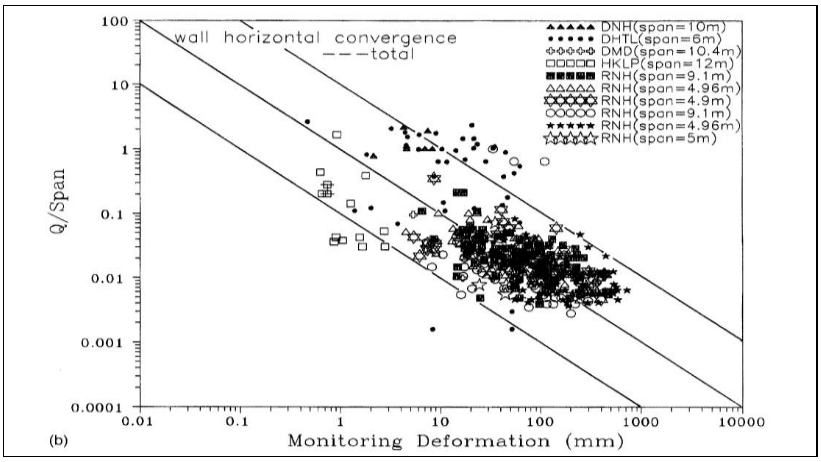

Fig. 9. *The central (very approximate) data trend of tunnel deformation versus span, modified by rock mass quality Q, can be described by the simplest equation that is possible in rock engineering. See Table 3 (left side).*

Recent reviews of pre-excavation modelling for cavern design, and actual cavern performance review for a major metro constructor in Asia, suggest that it is wise to consult these two simple equations, when deliberating over the reality (or not) of numerical models. It is the experience of the writer that distinct element UDEC-MC and UDEC-BB modellers often exaggerate the continuity of modelled jointing (because this is easier than drawing a more representative image of the less-continuous jointing, and digitising the latter). This may result in an order of magnitude error in deformation estimates.

A Fundamental Geotechnical Over-sight?

This paper will be concluded with a subject that concern the transformation of stress from a principal (2D) stress state of σ_1 and σ_2 to an inclined joint, fault or failure plane, to derive the commonly required *shear and normal stress components τ and σ_n*. If the surface onto which stress is to be transformed does not dilate, which might be the case with a (residual-strength) fault or clay-filled discontinuity, then the assumption of co-axial or co-planar stress and strain is no doubt valid. In general this and other assumptions are not valid.

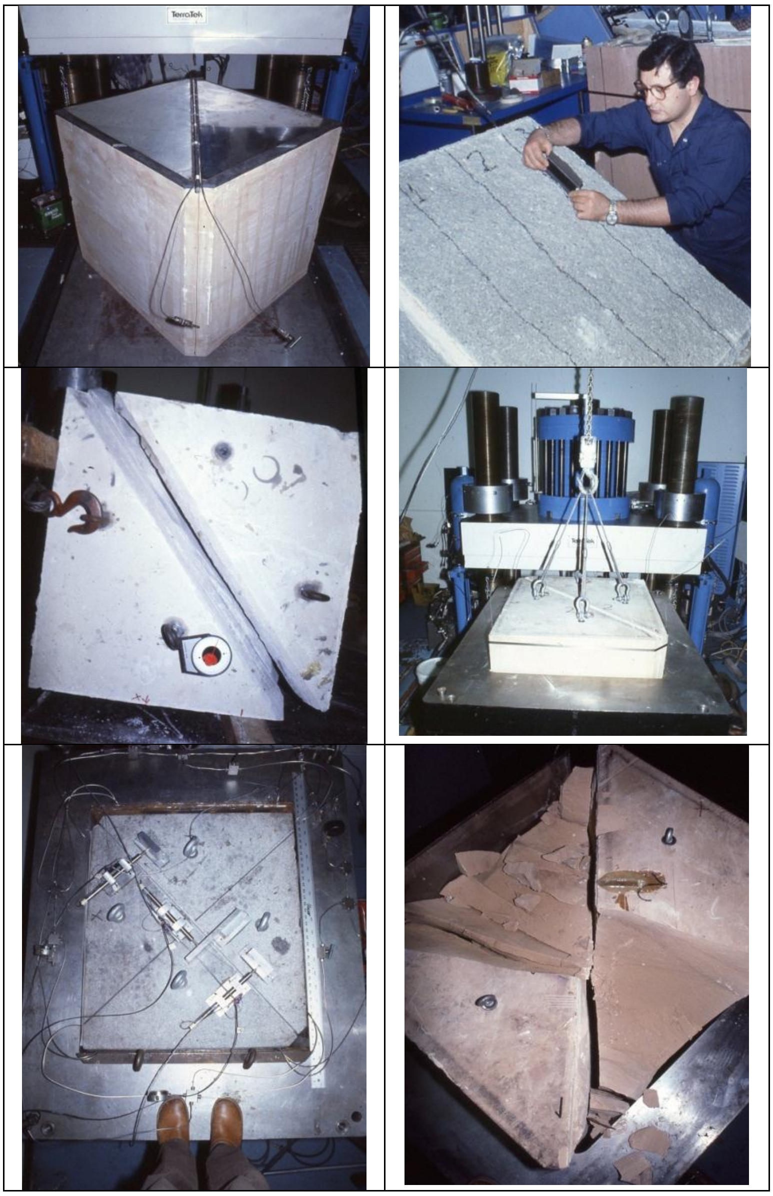

Fig. 10. *Sample preparation, roughness profiling, tilt testing (at 1 m^3 scale), lowering lightly clamped sample into test frame, LVDT instrumentation, and (a rare) sheared sample. The difficulty of shearing is due to an ignored aspect of stress transformation.*

CONVENTIONAL

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{1}{2}(\sigma_1 - \sigma_2)\cos 2\beta$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\beta$$

MODIFIED

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{1}{2}(\sigma_1 - \sigma_2)\cos 2[\beta + d_n \text{ mob}]$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_2)\sin 2[\beta + d_n \text{ mob}]$$

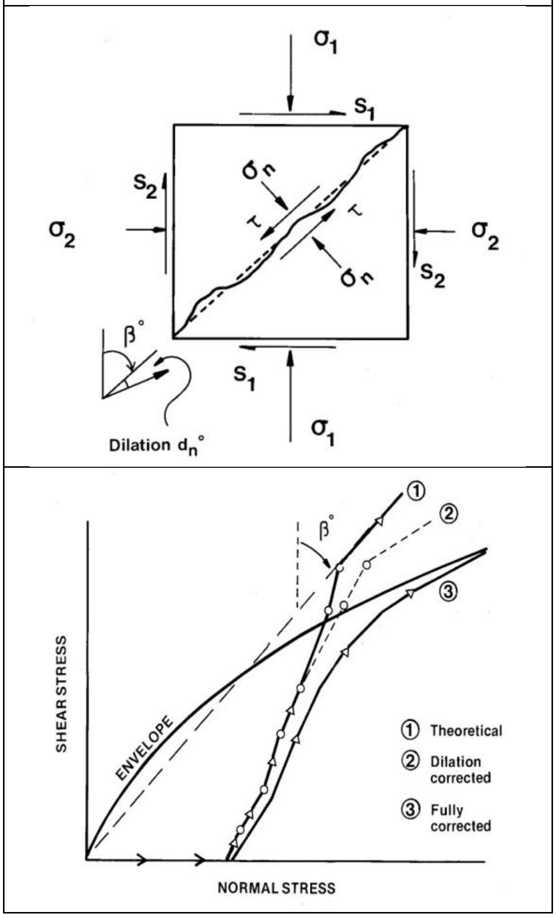

Fig. 11. *Corrections for out-of-plane dilation and boundary friction, after Bakhtar and Barton, 1984.*

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